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SIMPLE APPROXIMATE FORMULA FOR MUTUAL ADMITTANCE BETWEEN SLOTS ON A CYLINDER

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1. INTRODUCTION

This paper contains two results for the mutual admittance Y_{12} between two slots on the surface of a large conducting cylinder (including the conducting plane as a special case). The first and the main result is that an approximate, closed-form solution of Y_{12} is derived. This solution may be considered as a simplified version of the asymptotic solution of Y_{12} reported in [1], as the two surface integrals over the apertures of the slots are no longer needed in the present approximate solution. Our second result concerns the derivation of an exact solution of Y_{12}, which is given in terms of an inverse Fourier transform and an infinite summation of cylindrical modes. This solution is based on the original expression for Y_{12} described by Stewart, Golden, and Pridmore-Brown [2], [3], and is more suitable for numerical calculation for some cases.

This work is undertaken for the following reasons. The determination of Y_{12} (or its dual problem for Z_{12} between two dipoles) is not only a classical problem in electromagnetics that has attracted wide attention [1] - [10], but also an integral part in the design of modern conformal arrays [11] - [15]. In the latter application, Y_{12} must be repeatedly calculated for a large number of times. Thus, a simple closed-form solution should greatly reduce the computation effort and, furthermore, provide a better physical insight for the design problem as the "cause" and "effect" can be readily identified in a closed-form solution.

The organization of this paper is as follows. In Section 2, we first define Y_{12}, and then give the final form of its approximate solution. Discussions and numerical results are presented in Section 3. In the

last two sections (4 and 5), the derivations of both the approximate and the exact modal solutions of Y_{12} are given. Fock functions used in the text are described in the Appendix.

2. APPROXIMATE FORMULA FOR MUTUAL ADMITTANCE

Referring to Figure 1, consider two slots on the surface of an infinitely long conducting cylinder with radius R. The orientation of the slots may be either circumferential (Figure 1b where $a_n > b_n$, $n = 1,2$), or axial (Figure 1c where $a_n < b_n$). The problem is to determine the mutual admittance between these two slots when kR is large.

First let us define mutual admittance. Throughout this work we always assume that

(i) the slots are thin, and (2.1a)

(ii) their length is roughly a half-wavelength. (2.1b)

Then the aperture field in each slot can be adequately approximated by a simple cosine distribution, which is the so-called "one-mode" approximation. For example, if slot 1 is circumferential (Figure 1b), its aperture field under the "one-mode" approximation is given by

$$\vec{E} = V_1 \vec{e}_1 \ , \ \vec{H} = I_1 \vec{h}_1 \tag{2.2a}$$

where

$$\vec{e}_1 = \hat{z}\sqrt{\frac{2}{a_1 b_1}} \cos \frac{\pi}{a_1} y \quad , \quad \vec{h}_1 = \hat{x} \times \vec{e}_1 \tag{2.2b}$$

$$y = R\phi \ . \tag{2.2c}$$

(V_1, I_1) are respectively the modal (voltage, current) of slot 1. The mutual admittance Y_{12} is defined by

$$Y_{12} = Y_{21} = \frac{I_{21}}{V_1} \tag{2.3}$$

where I_{21} is the induced current in slot 2 when slot 1 is excited by a voltage V_1 and slot 2 is short-circuited. An alternative expression for Y_{12} is

$$Y_{12} = \frac{1}{V_1 V_2} \iint_{A_2} \vec{E}_2 \times \vec{H}_1 \cdot d\vec{s}_2 \tag{2.4}$$

where

A_2 = aperture of slot 2

$\vec{H}_1$ = magnetic field when slot 1 is excited with voltage V_1, and slot 2 is covered by a perfect conductor

$\vec{E}_2$ = electric field when slot 2 is excited with voltage V_2, and slot 1 is covered by a perfect conductor.

Because $\vec{H}_1 = I_{21}\vec{h}_2$ and $\vec{E}_2 = V_2\vec{e}_2$, it is a simple matter to verify that (2.3) and (2.4) are equivalent [16].

There is an alternative definition of mutual admittance. Instead of (2.2), a modal voltage $\bar{V}_1$ (with a bar) may be defined through the expression for the aperture field of slot 1 as follows:

$$\vec{E} = \hat{z}\,\frac{1}{b}\,\bar{V}_1 \cos\frac{\pi}{a_1} y \tag{2.5a}$$

or equivalently

$$\bar{V}_1 = \int_0^b (\hat{z} \cdot \vec{E})_{y=0}\, dz \quad . \tag{2.5b}$$

Then a different mutual admittance $\bar{Y}_{12}$ is defined by (2.4) after replacing (V_1, V_2) by $(\bar{V}_1, \bar{V}_2)$. It can be easily shown that

$$\bar{Y}_{12} = \frac{1}{2}\left(\frac{a_1 a_2}{b_1 b_2}\right)^{1/2} Y_{12} \quad . \tag{2.6}$$

Two remarks are in order: (i) In the limiting case that b_1 and $b_2 \rightarrow 0$, Y_{12} goes to zero as $(b_1 b_2)^{1/2}$, whereas $\bar{Y}_{12}$ approaches a constant independent of b_1 and b_2. (ii) For the special case $a_1 = a_2 = \lambda/2$ and

$R \to \infty$, it is $\bar{Y}_{12}$, not Y_{12}, that is identical to the mutual impedance Z_{12} between two corresponding dipoles calculated by the classical Carter's method [5], [8], [9]. (iii) When the slots are excited by waveguides (transmission lines), one often uses Y_{12} ($\bar{Y}_{12}$). From here on, we will concentrate on Y_{12} instead of $\bar{Y}_{12}$.

For the two slots in Figure 1, the final form of an approximate solution of Y_{12} is as follows (for exp $+j\omega t$ time convention):

Circumferential slots

$$Y_{12} \simeq -\frac{8}{\pi^2}(a_1 b_1 a_2 b_2)^{1/2}\, S(b_1 \sin\theta)\, S(b_2 \sin\theta)\, C(a_1 \cos\theta)\, C(a_2 \cos\theta)\, \bar{g}_\phi \tag{2.7a}$$

Axial slots

$$Y_{12} \simeq -\frac{8}{\pi^2}(a_1 b_1 a_2 b_2)^{1/2}\, S(a_1 \cos\theta)\, S(a_2 \cos\theta)\, C(b_1 \sin\theta)\, C(b_2 \sin\theta)\, \bar{g}_z\ . \tag{2.7b}$$

The various factors in (2.7) are explained below. S and C are simple trigonometric functions

$$S(x) = \frac{\sin(kx/2)}{(kx/2)}\ ,\quad C(x) = \frac{\cos(kx/2)}{1-(kx/\pi)^2}\ . \tag{2.8}$$

The (simplified) Green's functions $\bar{g}_\phi$ and $\bar{g}_z$ are given by

$$\bar{g}_\phi = G(s)\Big[v(\xi)\left(\sin^2\theta + \frac{j}{ks}\cos 2\theta\right) + \frac{j}{ks}\,u(\xi)\cos^2\theta$$
$$+\ ju'(\xi)(\sqrt{2}\,kR\cos\theta)^{-2/3}\sin^4\theta\Big] \tag{2.9a}$$

$$\bar{g}_z = G(s)\Big[v(\xi)\left(\cos^2\theta - \frac{j}{ks}\cos 2\theta\right) + \frac{j}{ks}\,u(\xi)\sin^2\theta\Big] \tag{2.9b}$$

where

$$G(s) = \frac{k^2 Y_0}{2\pi j}\frac{e^{-jks}}{ks}\ ,\quad Y_0 = \frac{1}{120\pi} \tag{2.10}$$

$$\xi = (k\cos^4\theta/2R^2)^{1/3}\, s \tag{2.11}$$

$$s = \sqrt{z_0^2 + (R\phi_0)^2} \tag{2.12}$$

$$\theta = \tan^{-1} (z_0/R\phi_0) \ . \tag{2.13}$$

The Fock functions u and v are explained in the Appendix. In the limiting case $kR \to \infty$ (slots on a planar surface), (2.9) is further simplified to become

$$\left.\begin{aligned} \bar{g}_\phi &= G(s) \left[\sin^2 \theta + \frac{j}{ks} (2 - 3 \sin^2 \theta)\right] , \\ \bar{g}_z &= G(s) \left[\cos^2 \theta + \frac{j}{ks} (2 - 3 \cos^2 \theta)\right] , \end{aligned}\right. \quad kR \to \infty \ . \tag{2.14}$$

The formula in (2.4) is an approximate solution, valid under the condition

$$kR >> 1 \quad \text{and} \quad ks >> 1 \ . \tag{2.15}$$

The numerical accuracy of the formula is discussed in Section 3, and its derivation in Section 4.

3. NUMERICAL RESULTS AND DISCUSSION

For the two slots in Figure 1, the final form of the approximate solution of Y_{12} is given in (2.7). Generally speaking, its accuracy is good only if

(i) the size of the slots is small in terms of wavelength, and/or

(ii) the separation of the slots is large in terms of wavelength.

In this section, we will give some numerical examples to illustrate the quantitative accuracy of (2.7).

(A) Circumferential Slot - (Figures 2 and 3). The size of each slot is $0.5\lambda \times 0.2\lambda$, and the cylinder radius is 1λ. Y_{12} is presented in (dB, normalized phase) format, where $dB = 20 \log_{10} (|Y_{12}|$ in mho) and normalized phase is equal to $\mathrm{Arg}(Y_{12}\ \mathrm{exp} jks)$. Three solutions of Y_{12} are given: the UI exact modal solution calculated from (5.2), (5.3) and (5.9); the UI asymptotic solution reported in [1]; and the approximate solution in (2.7). We note that all the three solutions are in an excellent agreement.

(B) Percentage Error vs. Slot Position - (Figures 4 and 5). In these figures, the coordinates of each point determine the center-to-center distance, in ϕ and z directions between two slots. The pairs of numbers in the parentheses are the percentage error in magnitude and the absolute error in phase of Y_{12} as calculated by the approximate formula, respectively. For the circumferential slots (Figure 4), the accuracy is generally very good. For the axial slots (Figure 5), the approximate formula gives erratic results (as high as 27 percent error in magnitude) when the two slots are very closely displaced in the ϕ-direction. The reason for this inaccuracy is that the surface field due to a magnetic dipole varies very rapidly as a function of z when the observation point is close by.

(C) Accuracy vs. Cylinder Radius (Figure 6). The accuracy of the approximate formula is not sensitive to the radius of the cylinder.

(D) Planar Slots (Tables 1 and 2). The mutual admittance Y_{12} between two identical slots of dimension ($a = 0.69\lambda$, $b = 0.3\lambda$) on an infinite conducting plane is calculated as a function of z_0 and y_0 (the center-to-center distance between two slots in z and y directions, see Figure 1b). Y_{12} is given in (dB, phase in degrees). In both E-plane and H-plane couplings, the approximate formula is accurate when the separation is at least two wavelengths (2.6"). It should be also remarked that the present slots ($0.69\lambda \times 0.3\lambda$) are relatively large. The accuracy of the approximate formula is better when the slots are smaller.

4. DERIVATION OF APPROXIMATE FORMULA

We will now give the derivation of the formula in (2.7a)[that of (2.7b) is very similar]. Consider a circumferential infinitesimal dipole located at Q' on the surface of a cylinder (Figure 7) which is described by the magnetic current density

$$\vec{K} = \hat{\phi}\,\frac{1}{R}\,\delta(r - R)\,\delta(\phi)\,\delta(z) \quad . \tag{4.1}$$

At an observation point Q on the cylinder, the ϕ-component of the $\vec{H}$ field, denoted by g_ϕ, is determined in Eq. (2.16b) of [1], which reads in the present notation,

$$\begin{aligned} g_\phi(t,\alpha) \sim G(t) \Bigg\{ & v(\xi)\left[\sin^2\alpha + \frac{j}{kt}\cos 2\alpha\right] \\ & + \left(\frac{j}{kt}\right) u(\xi)\left[\cos^2\alpha\left(1 - \frac{2j}{kt}\right) + \left(\frac{j}{kt}\right)\sin^2\alpha\right] \\ & + j(\sqrt{2}\,kR/\cos^2\alpha)^{-2/3} \\ & \cdot \left[v'(\xi)\sin^2\alpha + \left(\tan^4\alpha + \frac{j}{kt}\right)u'(\xi)\cos^2\alpha\right]\Bigg\} \end{aligned} \tag{4.2}$$

where (t,α) are the cylindrical coordinates of Q with respect to the origin at Q' on a developed cylinder, and

$$\xi = (k\cos^4\theta/2R^2)^{1/3}\,t \quad . \tag{4.3}$$

The formula in (4.2) is mainly based on a classical work of Fock [17], and contains a modification that introduces a field dependence on the surface curvature in the binormal direction of the surface ray (see Section 6 of [1]). This formula is asymptotically valid for $kR \to \infty$, and may be used to calculate the field at any point on the cylindrical surface.

Making use of the Green's function in (4.2), we next calculate the surface field H_ϕ due to slot 1 on a cylinder (Figure 8). The aperture distribution of slot 1 is described in (2.2a), which may be replaced by an equivalent magnetic current density (p. 108 of [18])

$$\vec{K} = \hat{\phi}\,\delta(r - R)\sqrt{\frac{2}{a_1 b_1}}\,V_1 \cos(\pi y/a_1) \quad . \tag{4.4}$$

Then, H_ϕ at an observation point Q is obtained by superposition, namely,

$$H_\phi(Q) = \sqrt{\frac{2}{ab}}\,V_1 \iint_{A_1} \left(\cos\frac{\pi}{a_1}y\right) g_\phi(t,\alpha)\,dy\,dz \quad . \tag{4.5}$$

The expression for calculating the mutual admittance Y_{12} between the two slots in Figure 8 is given in (2.4). Note that $\vec{E}_2$ is described much as (2.2a) and $\vec{H}_1$ in (4.5). Then (2.4) becomes

$$Y_{12} = \frac{-2}{\sqrt{a_1 b_1 a_2 b_2}} \iint_{A_1} dy\,dz \iint_{A_2} dy_2\,dz_2 \left(\cos\frac{\pi}{a_1}y\right)\left(\cos\frac{\pi}{a_2}y_2\right) g_\phi(t,\alpha) \quad . \tag{4.6}$$

The distance t in (4.6) is given by

$$t = [(s\cos\theta + y_2 - y)^2 + (s\sin\theta + z_2 - z)]^{1/2} \quad . \tag{4.7}$$

If s is large relative to the length of either slot, t may be approximated by

$$t \simeq \begin{cases} s & (4.8a) \\ s\left(1 + \cos\theta\,\dfrac{y_2 - y}{s} + \sin\theta\,\dfrac{z_2 - z}{s}\right) & (4.8b) \end{cases} \quad .$$

In evaluating the magnitude of g_ϕ in (4.6), we use the approximation in (4.8a), whereas in evaluating its progressive phase term, we use (4.8b).

Then the integrals in (4.6) can be explicitly carried out. After a further approximation by dropping the terms of order $(ks)^{-3} = (kt)^{-3}$ in (4.2), we obtain the desired solution of Y_{12} in (2.7a).

5. EXACT MODAL SOLUTION

The admittance Y_{12} defined in (2.3) may be calculated exactly by using cylindrical modes, as has been done by Stewart, Golden and Pridmore-Brown [2], [3]. Extensive numerical results of Y_{12} calculated from the SGP solution are reported in [13], [14]. As will be explained below, the SGP solution is not suitable for numerical calculations when the slot separation z_0 (Figure 1a) is large. In this section, we will derive an alternative modal solution of Y_{12} which does not have this difficulty.

Let us first consider the circumferential slots shown in Figure 1b. For the case that $a_1 = a_2 = a$ and $b_1 = b_2 = b$ (identical slots), the mutual admittance Y_{12} is given in Eq. (8) of [3]*, which reads in the present notation,

$$Y_{12} = \int_{-\infty}^{\infty} dk_z \sum_{m=-\infty}^{\infty} \psi(m,k_z) G(m,k_z) e^{-j(m\phi_0 + k_z z_0)} \tag{5.1a}$$

where

$$\psi(m,k_z) = \frac{ab}{8\pi^2 R} \frac{\sin^2(k_z b/2)}{(k_z b/2)^2} \cdot \left\{ \frac{\sin(m\phi_a + \pi/2)}{(m\phi_a + \pi/2)} + \frac{\sin(m\phi_a - \pi/2)}{(m\phi_a - \pi/2)} \right\}^2 \tag{5.1b}$$

$$\phi_a = (a/2R)$$

$$G(m,k_z) = Y_0 \left[\frac{jk}{k_t} \frac{H_m^{(2)'}(k_t R)}{H_m^{(2)}(k_t R)} + \left(\frac{mk_z}{k_t^2 R} \right)^2 \frac{k_t}{jk} \frac{H_m^{(2)}(k_t R)}{H_m^{(2)'}(k_t R)} \right] . \tag{5.1c}$$

---

* The multiplication factor 2 in the definition of ϕ_b in [3] is a misprint and should be removed.

$$k_t = \begin{cases} \sqrt{k^2 - k_z^2} & , \text{ if } k \geq k_z \\ -j\sqrt{k_z^2 - k^2} & , \text{ if } k \leq k_z \end{cases} .$$

Rewrite Y_{12} in terms of its real and imaginary parts:

$$Y_{12} = G + jB \quad . \tag{5.2}$$

It can be shown that G is given by

$$G = \int_0^k \sum_{m=0}^{\infty} \frac{\cos m\phi_0}{\varepsilon_m} \cos k_z z_0 \; \psi(m,k_z) R(m,k_z) \; dk_z \tag{5.3a}$$

where

$$R(m,k_z) = \frac{2}{\pi k_t R} \cdot \frac{k}{k_t} \cdot \left[\frac{1}{M_m^2(k_t R)} + \left(\frac{mk_z}{k_t k R} \right)^2 \frac{1}{N_m^2(k_t R)} \right] \tag{5.3b}$$

$$M_m^2(\chi) = J_m^2(\chi) + Y_m^2(\chi) \tag{5.3c}$$

$$N_m^2(\chi) = J_m'^2(\chi) + Y_m'^2(\chi) \tag{5.3d}$$

$$\varepsilon_m = \begin{cases} 2 \; , \; m = 0 \\ 1 \; , \; m \neq 0 \end{cases} . \tag{5.3e}$$

We note that G contains a *finite* integral and can be evaluated in a straightforward manner by standard numerical integration techniques. The imaginary part of Y_{12} is given by

$$B = \int_{C_1} \sum_{m=0}^{\infty} \frac{\cos m\phi_0}{\varepsilon_m} \cdot \cos k_z z_0 \cdot \psi(m,k_z) \cdot W(m,k_z) \; dk_z \tag{5.4a}$$

where the integration contour C_1 is shown in Figure 9 and

$$W(m,k_z) = \begin{cases} \dfrac{k}{k_t}\,(J_m J_m' + Y_m Y_m')\left[\dfrac{1}{M_m^2(k_t R)} - \left(\dfrac{mk_z}{k_t k R}\right)^2 \dfrac{1}{N_m^2(k_t R)}\right], & \text{if } k > k_z \\[2ex] \dfrac{-k}{|k_t|}\left[\dfrac{K_m'(|k_t|R)}{K_m(|k_t|R)} - \left(\dfrac{mk_z}{|k_t|kR}\right)^2 \dfrac{K_m(|k_t|R)}{K_m'(|k_t|R)}\right], & \text{if } k < k_z \end{cases} \tag{5.4b}$$

The computation of B as given in (5.4a) can be quite laborious because (i) the integration with respect to k_z is of infinite range, and the factor $\cos k_z z$ is highly oscillatory for large kz_0, (ii) $W(m,k_z)$ has nonintegrable singularities of opposite sign on both sides of $k_z = k$ (iii) $W(m,k_z)$ decays slowly with respect to m and k_z.

To circumvent the above difficulties in evaluating B, we adopt a method introduced by Duncan [19] in the study of cylindrical antenna problems. Let us rewrite (5.4a)

$$B = \mathrm{Im}\left\{\sum_{m=0}^{\infty} \frac{\cos m\phi_0}{\varepsilon_m}\left[-j\int_{C_1} F(m,k_z)\sin k_z z_0\, dk_z + \int_{C_1} F(m,k_z)e^{jk_z z_0}\, dk_z\right]\right\} \tag{5.5}$$

where

$$F(m,k_z) = [R(m,k_z) + jW(m,k_z)]\psi(m,k_z) \quad . \tag{5.6}$$

The imaginary part of the first term inside the bracket of (5.5) is

$$\mathrm{Im}\left\{-j\int_{C_1} F(m,k_z)\sin k_z z_0\, dk_z\right\} = -\int_0^k R(m,k_z)\psi(m,k_z)\sin k_z z_0\, dk_z \quad . \tag{5.7}$$

In order to compute the imaginary part of the second term of (5.5), the integration contour C_1 is deformed into C_2 (Figure 9) according to the theory of complex variables. This manipulation leads to

$$\mathrm{Im}\int_{C_1} F(m,k_z)e^{jk_z z_0}\, dk_z = \mathrm{Im}\int_{C_2} F(m,k_z)e^{jk_z z_0}\, dk_z \quad . \tag{5.8}$$

Make the change of variable $k_z = j\eta$ in (5.8). Substitution of the resultant equation and (5.7) into (5.5) gives

$$B = \sum_{m=0}^{\infty} \frac{\cos m\phi_0}{\varepsilon_m} \left\{ -\int_0^k R(m,k_z)\psi(m,k_z) \sin k_z z_0 \, dk_z \right. \tag{5.9}$$
$$\left. + \int_0^{\infty} R(m,j\eta)\psi(m,j\eta) e^{-\eta z_0} \, d\eta \right\} .$$

Our final expression for Y_{12} is given in (5.2), with its real part G in (5.3) and its imaginary part B in (5.9). Several remarks are in order: (i) Not only G but also B is determined by $R(m,k_z)$, which is much simpler than $W(m,k_z)$ defined in (5.4b). (ii) B contains only a finite integral. (iii) The infinite integral in B, i.e., the second integral in (5.9a), contains an exponentially decaying factor $\exp[-z_0 - a)\eta]$ in its integrand. The emergence of the evaluation of B is faster for larger z_0. This is in contrast to the original expression of Y_{12} given in (5.1). (iv) There is no nonintegrable singularity in (5.3) or (5.9).

The same method applies to the derivation of an alternative expression of Y_{12} for two identical axial slots (Figure 1c with $a_1 = a_2 = a$ and $b_1 = b_2 = b$). We give below only the final result:

$$Y_{12} = -\frac{abY_0}{\pi k R^2} \sum_{m=0}^{\infty} \frac{\cos m\phi_0}{\varepsilon_m} \left[\int_0^k \Phi(m,k_z) e^{-jk_z z_0} \frac{dk_z}{N_m^2(k_t R)} \right. \tag{5.10a}$$
$$\left. + j \int_0^{\infty} \Phi(m,j\eta) e^{-\eta z_0} \frac{d\eta}{N_m^2\left(R\sqrt{\eta^2 + k^2}\right)} \right]$$

where

$$\Phi(m,k_z) = \left[\frac{\sin(m\phi_a)}{(m\phi_a)} \cdot \frac{\cos(k_z b/2)}{(k_z b/2)^2 - (\pi/2)^2} \right]^2 . \tag{5.10b}$$

In summary, the alternative expression of the exact modal solutions is given in (5.2), (5.3), and (5.9) for two identical circumferential slots, and in (5.10) for two identical axial slots.

APPENDIX

FOCK FUNCTIONS

In this appendix we define and list some useful formulas of the functions $w_1(t)$, $w_2(t)$, $v(\xi)$, $u(\xi)$, and $v_1(\xi)$. These functions are commonly known as Fock functions.

(i) Definition: For a complex t and a real ξ,

$$w_1(t) = \frac{1}{\sqrt{\pi}} \int_{\Gamma_1} dz \exp\left(tz - \frac{1}{3} z^3\right) \tag{A-1}$$

$$w_2(t) = \frac{1}{\sqrt{\pi}} \int_{\Gamma_2} dz \exp\left(tz - \frac{1}{3} z^3\right) = w_1^*(t) \tag{A-2}$$

$$v(\xi) = \frac{1}{2} e^{j\pi/4} \xi^{1/2} \frac{1}{\sqrt{\pi}} \int_{\Gamma_1} \frac{w_2(t)}{w_2'(t)} e^{-j\xi t} dt \tag{A-3}$$

$$u(\xi) = e^{j3\pi/4} \xi^{3/2} \frac{1}{\sqrt{\pi}} \int_{\Gamma_1} \frac{w_2'(t)}{w_2(t)} e^{-j\xi t} dt \tag{A-4}$$

$$v_1(\xi) = e^{j3\pi/4} \xi^{3/2} \frac{1}{\sqrt{\pi}} \int_{\Gamma_1} t \frac{w_2(t)}{w_2'(t)} e^{-j\xi t} dt \tag{A-5}$$

where integration contour $\Gamma_1 (\Gamma_2)$ goes from ∞ to 0 along the line $\operatorname{Arg} z = -2\pi/3$ $(+2\pi/3)$ and from 0 to ∞ along the real axis. Because of different time conventions, $w_1 (w_2)$ above is equal to $w_2 (w_1)$ defined in [17].

(ii) Residue series representation: For real positive ξ,

$$v(\xi) = e^{-j\pi/4}\sqrt{\pi}\ \xi^{1/2} \sum_{n=1}^{\infty} (t_n')^{-1} e^{-j\xi t_n'} \tag{A-6}$$

$$u(\xi) = e^{j\pi/4} 2\sqrt{\pi}\ \xi^{3/2} \sum_{n=1}^{\infty} e^{-j\xi t_n} \tag{A-7}$$

$$v_1(\xi) = e^{j\pi/4} 2\sqrt{\pi}\ \xi^{3/2} \sum_{n=1}^{\infty} e^{-j\xi t_n'} \tag{A-8}$$

$$v'(\xi) = \frac{1}{2} e^{-j\pi/4}\sqrt{\pi}\ \xi^{-1/2} \sum_{n=1}^{\infty} (1 - j2\xi t_n')(t_n')^{-1} e^{-j\xi t_n'} \tag{A-9}$$

$$u'(\xi) = e^{j\pi/4} 3\sqrt{\pi}\ \xi^{1/2} \sum_{n=1}^{\infty} \left(1 - j\frac{2}{3}\xi t_n\right) e^{-j\xi t_n} \tag{A-10}$$

where $\{t_n\}$ and $\{t_n'\}$ are zeros of $w_2(t)$ and $w_2'(t)$, respectively, and are tabulated in [17] and [1].

(iii) Small argument asymptotic expansion: For real positive ξ and $\xi \to 0$,

$$v(\xi) \sim 1 - \frac{\sqrt{\pi}}{4} e^{j\pi/4}\xi^{3/2} + \frac{7j}{60}\xi^3 + \frac{7\sqrt{\pi}}{512} e^{-j\pi/4}\xi^{9/2} - 4.141 \times 10^{-3}\xi^6 + \ldots \tag{A-11}$$

$$u(\xi) \sim 1 - \frac{\sqrt{\pi}}{2} e^{j\pi/4}\xi^{3/2} + \frac{5j}{12}\xi^3 + \frac{5\sqrt{\pi}}{64} e^{-j\pi/4}\xi^{9/2} - 3.701 \times 10^{-2}\xi^6 + \ldots \tag{A-12}$$

$$v_1(\xi) \sim 1 + \frac{\sqrt{\pi}}{2} e^{j\pi/4}\xi^{3/2} - \frac{7j}{12}\xi^3 - \frac{7\sqrt{\pi}}{64} e^{-j\pi/4}\xi^{9/2} + 4.555 \times 10^{-2}\xi^6 + \ldots \tag{A-13}$$

$$v'(\xi) \sim \frac{3\sqrt{\pi}}{8} e^{-j3\pi/4}\xi^{1/2} + \frac{7j}{20}\xi^2 + \frac{63\sqrt{\pi}}{1024} e^{-j\pi/4}\xi^{7/2} - 2.485 \times 10^{-2}\xi^5 + \ldots \tag{A-14}$$

$$u'(\xi) \sim \frac{3}{4}\sqrt{\pi}\ e^{-j3\pi/4}\xi^{1/2} + \frac{5j}{4}\xi^2 + \frac{45\sqrt{\pi}}{128} e^{-j\pi/4}\xi^{7/2} - 2.221 \times 10^{-1}\xi^5 + \ldots \tag{A-15}$$

(iv) Numerical evaluation: For $\xi \geq \xi_0$, the residue series representation with the first ten terms in the summation may be used. For $\xi \leq \xi_0$, the small argument asymptotic expansion with the first five terms may be used. It has been indicated in [12] that the smoothest crossover is obtained if $\xi_0 = 0.6$. In the present study, we set $\xi_0 = 0.7$, where the difference in the two representations is less than 0.1% in magnitude and 0.9° in phase [1].

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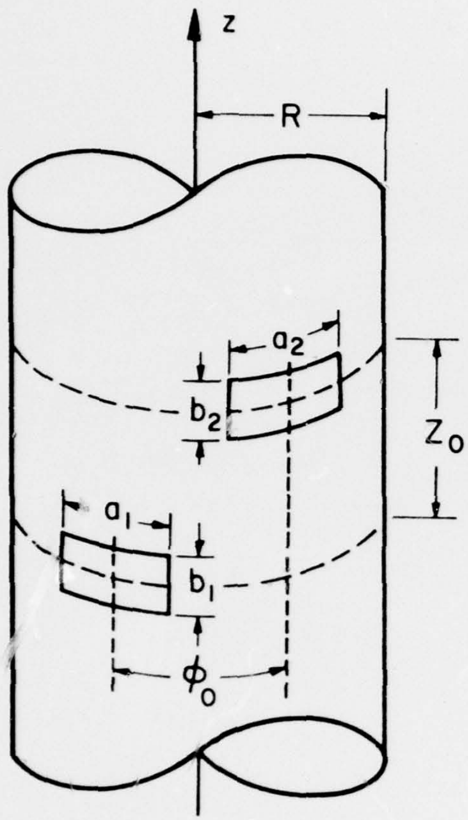
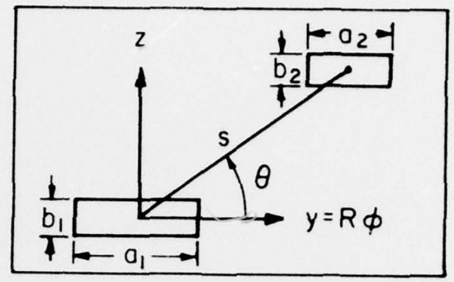
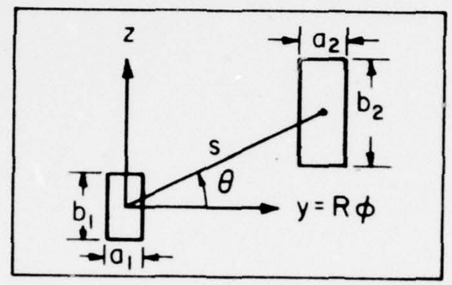


(a) Three-dimensional view



(b) Circumferential slots on developed cylinder.



(c) Axial slots on developed cylinder.

Figure 1. Two slots on the surface of a conducting cylinder.

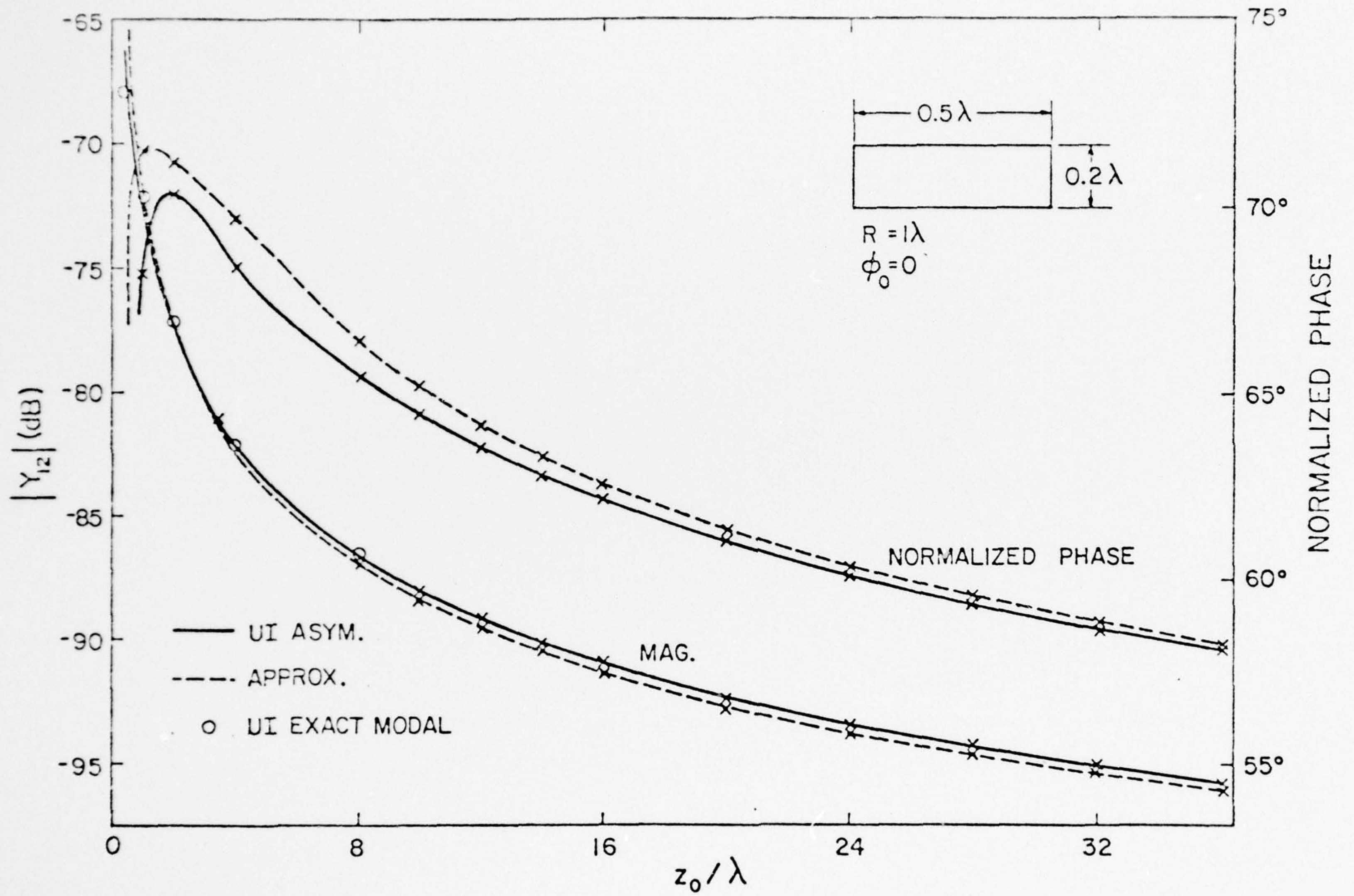


Figure 2. Mutual admittance Y_{12} between two circumferential slots as a function of z_0.

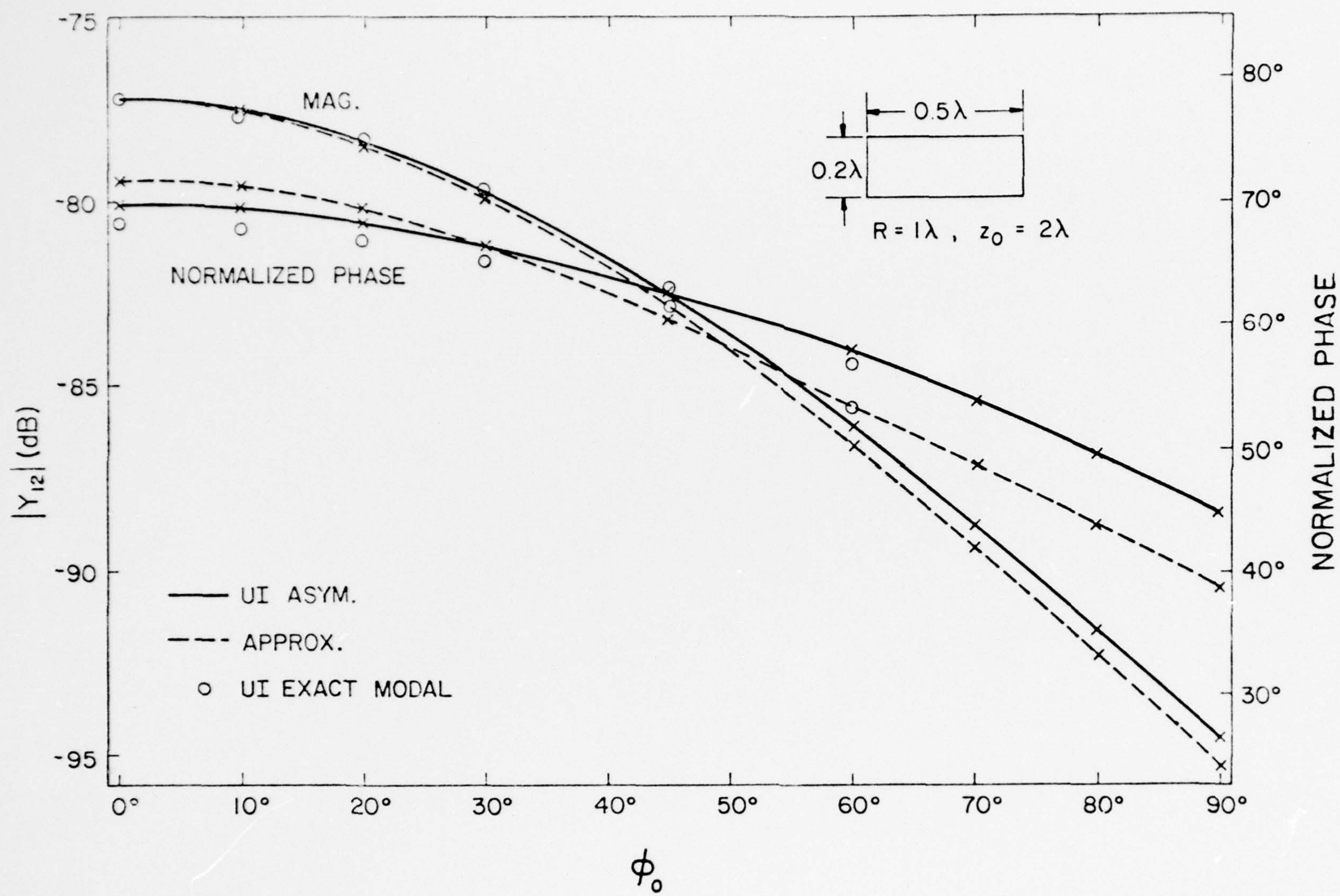


Figure 3. Mutual admittance Y_{12} between two circumferential slots as a function of ϕ_0.

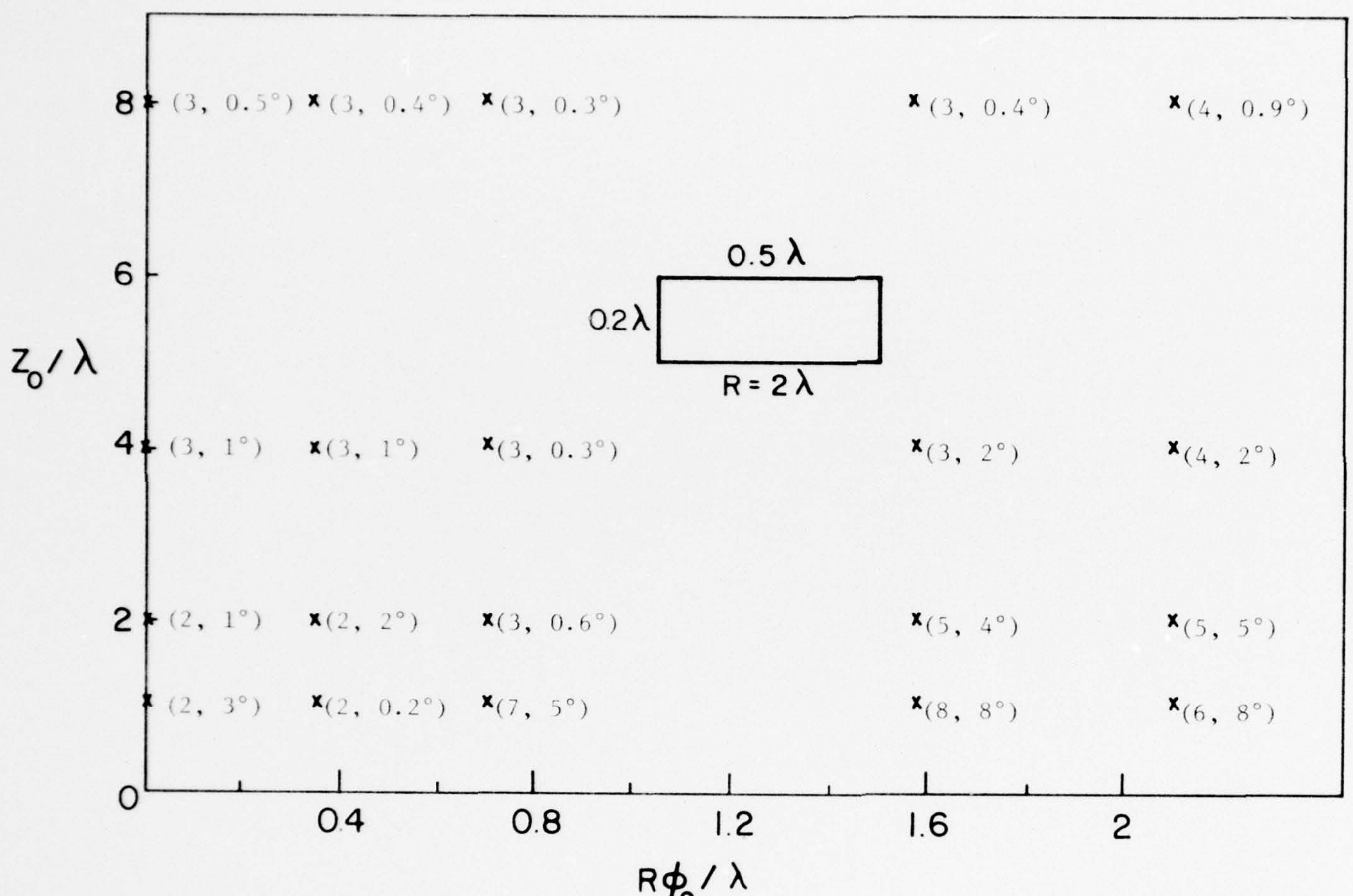


Figure 4. The percentage error in magnitude and absolute error in phase of the approximate formula of Y_{12} of circumferential slots as a function of their relative positions.

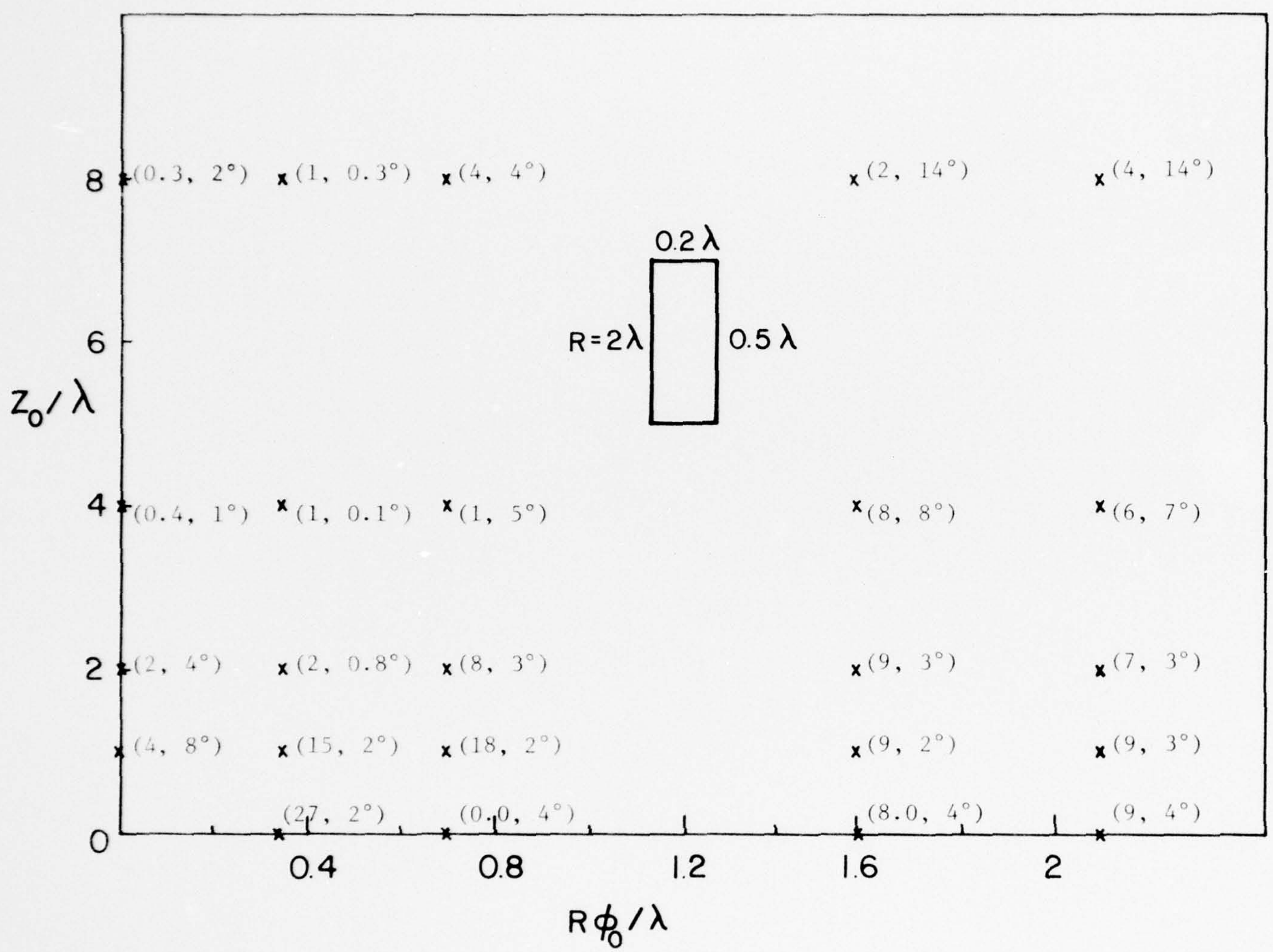


Figure 5. The percentage error in magnitude and absolute error in phase of the approximate formula of Y_{12} of axial slots as a function of their relative positions.

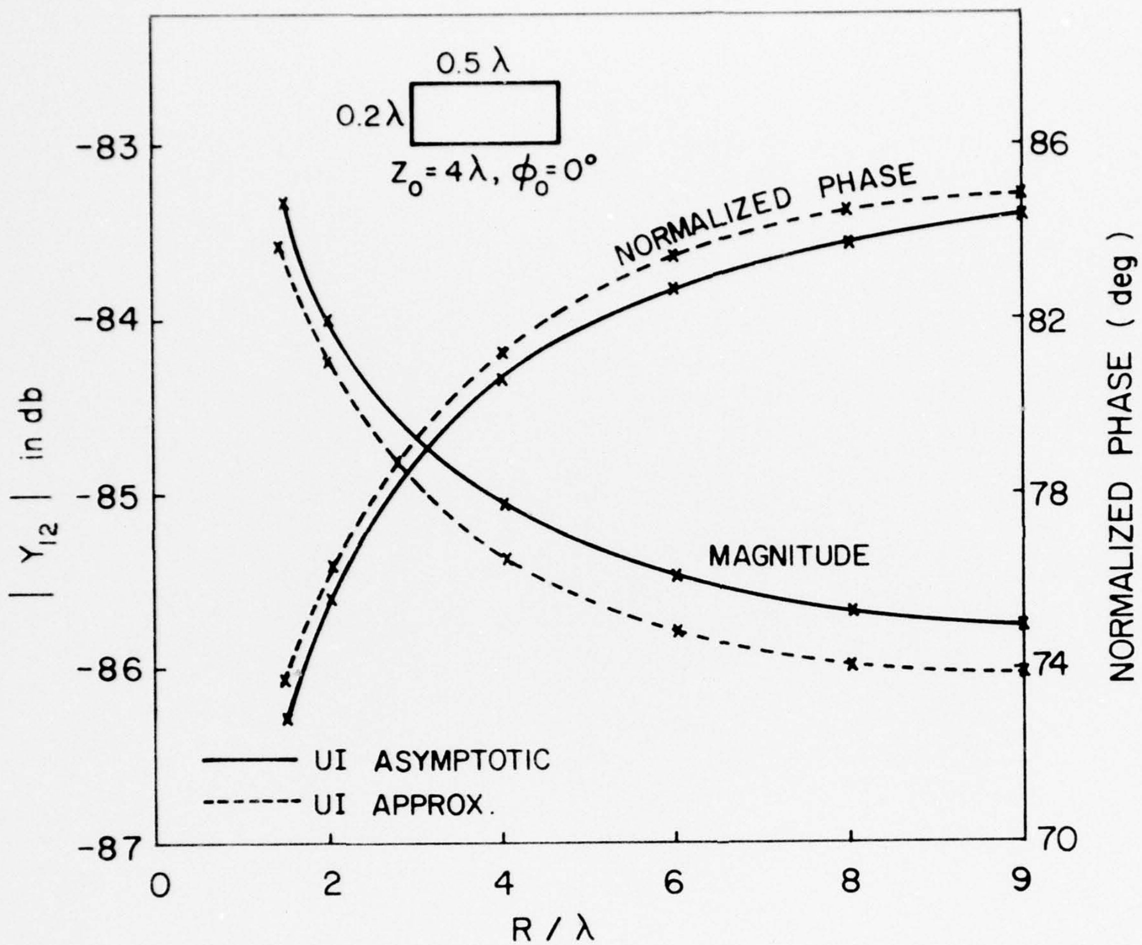
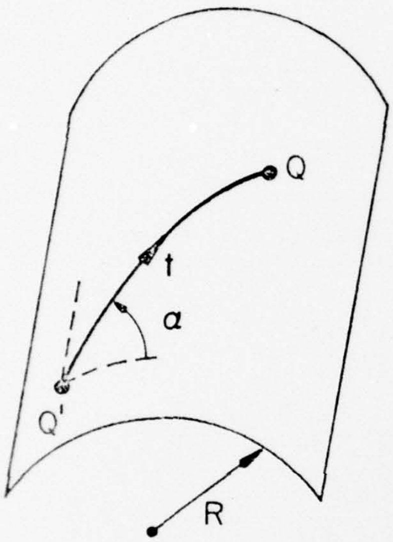
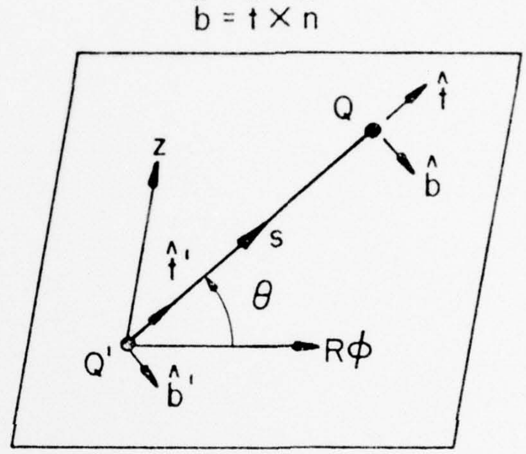


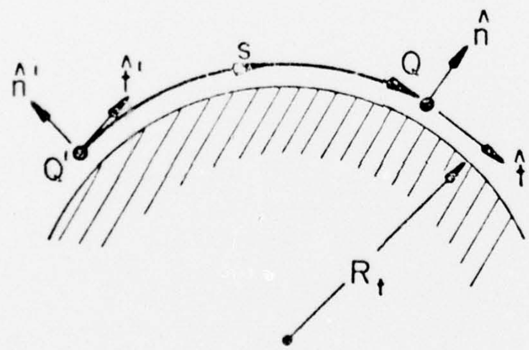
Figure 6. Mutual admittance Y_{12} between two identical circumferential slots as a function of radius R of the cylinder.



(a) 3-D view



(b) Developed cylinder



(c) Cut along θ-direction

Figure 7. A surface ray from source point Q' to observation point Q on a cylinder of radius R.

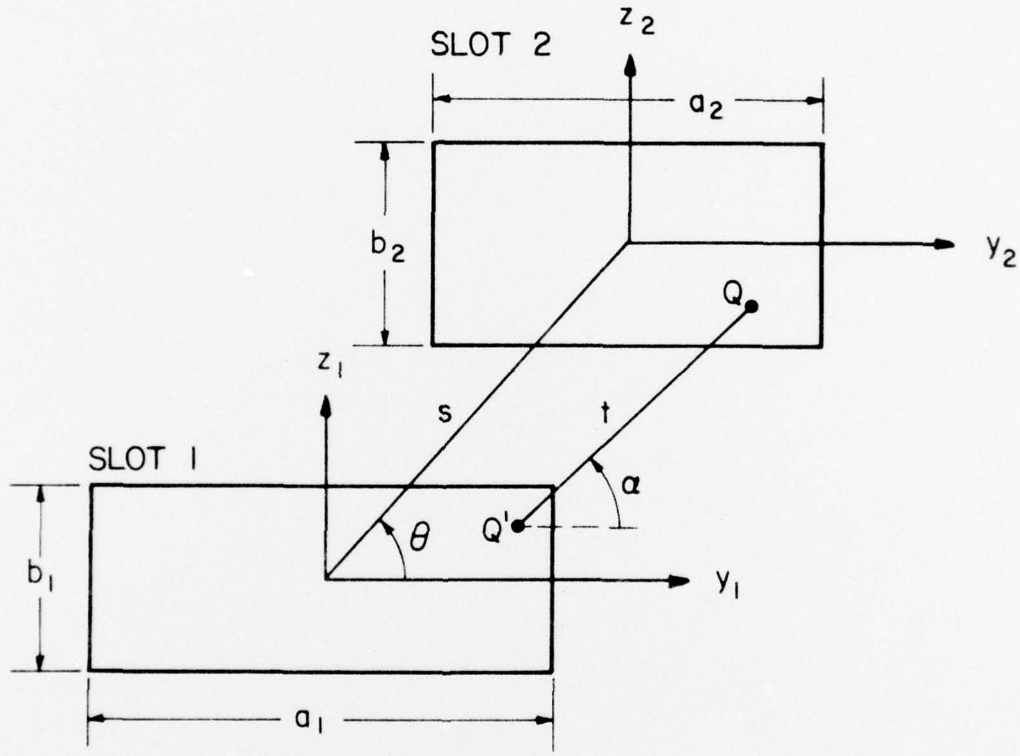


Figure 8. Two circumferential slots on a developed cylinder.

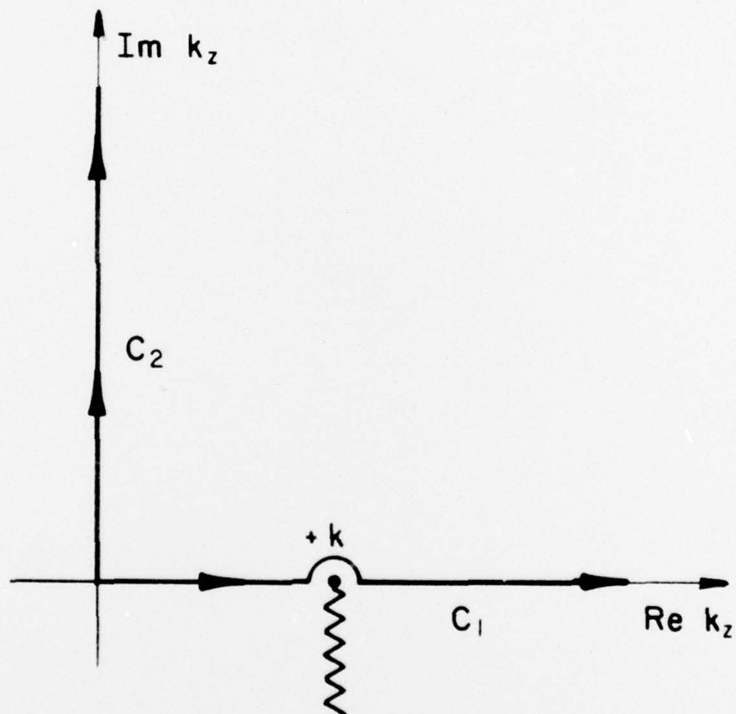


Figure 9. Contours in the complex k_z-plane for the integral in (5.4).

TABLE 1

MUTUAL ADMITTANCE Y_{12} BETWEEN TWO SLOTS

ON A PLANE (E-PLANE COUPLING)

z_0	Exact	Approximate
0.5λ	-64.57 dB -110°	-63.25 dB -108°
1λ	-69.48 78°	-69.58 81°
2λ	-75.13 84°	-75.68 85°
3λ	-78.58 86°	-79.22 87°
4λ	-81.06 87°	-81.72 88°
8λ	-87.05 88°	-87.75 89°

TABLE 2

MUTUAL ADMITTANCE Y_{12} BETWEEN SLOTS ON

A PLANE (H-PLANE COUPLING)

y_0	Exact	Approximate
1λ	-83.41 dB -53°	-85.04 dB -180°
2λ	-96.75 -168°	-97.09 -180°
3λ	-104.00 -172°	-104.13 -180°
4λ	-109.07 -174°	-109.13 -180°
8λ	-121.18 -177°	-121.17 -180°